AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA | DEPARTMENT OF EDUCATION AND SCIENCE

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Marking Scheme *Leaving Certificate Examination, 2000*

Applied Mathematics *Ordinary Level*

AN **ROINN OIDEACHAIS AGUS EOLAÍOCHTA**

LEAVING CERTIFICATE EXAMINATION, 2000

APPLIED MATHEMATICS – ORDINARY LEVEL

MARKING SCHEME

1. A car is travelling on a straight stretch of level road [pq]. The car passes the point p with a speed of 5 m/s and accelerates uniformly to its maximum speed of 20 m/s in a time of
6 seconds. The car continues with this maximum speed for 30 seconds before decelerating uniformly to rest at q in a further 4 seconds.

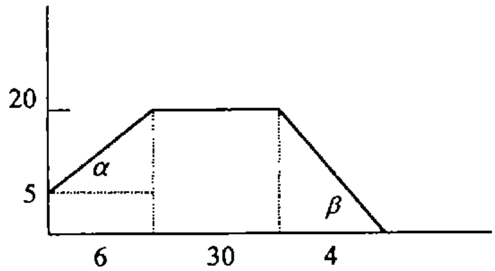
Draw a speed-time graph of the motion of the car from p to q.

Hence, or otherwise, find

(i) the uniform acceleration of the car

(ii) the uniform deceleration of the car

(iii) $|pq|$, the distance from p to q



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(i) $\tan\alpha = \frac{15}{6}$ or 2.5 — 10

(ii) $\tan\beta = \frac{20}{4}$ or 5 — 5

(iii) Distance $= 6(5) + \frac{1}{2}(6)(15) + 30(20) + \frac{1}{2}(4)(20)$

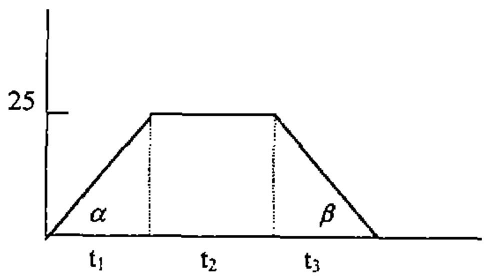
$= 30 + 45 + 600 + 40$

$= 715$ — 10

35

1 cont.

Another car, with acceleration and deceleration the same as in (i) and (ii) above. starts from rest at p and accelerates uniformly to its maximum speed of 25 m/s. It continues with this maximum speed for a certain time and then decelerates uniformly to rest at q.
How long does it take this car to go from p to q ?



$\tan\alpha = \frac{25}{t_1} \Rightarrow 2.5 = \frac{25}{t_1} \Rightarrow t_1 = 10$ 5

$\tan\beta = \frac{25}{t_3} \Rightarrow 5 = \frac{25}{t_3} \Rightarrow t_3 = 5$ 5

$\text{Distance} = \frac{1}{2}(10)(25) + 25t_2 + \frac{1}{2}(5)(25)$

$715 = 125 + 25t_2 + 62.5$

$t_2 = 21.1$

Total time $= 10 + 5 + 21.1 = 36.1\text{s}$ 5 | 15

2. **(a)** Ship A is n-welling with a speed of 15 km/hr in the direction due East.
Ship B is travelling with a speed of 20 km/hr in the direction due South.

Find the velocity of ship A relative to ship B.

(b) A river is 100 m wide and is flowing with a speed of 2 m/s parallel to the straight banks. The speed of a swimmer in still water is 3 m/s.

(i) What is the shortest time it takes the swimmer to swim across the river?

(a) What direction should the swimmer take so as to swim straight across to a point directly opposite?
How long will it then take the swimmer to cross to this point?

(a)

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$ or diagram — 10

$= 15\vec{i} - (-205)$

$= 15\vec{i} + 203$ — 10

or

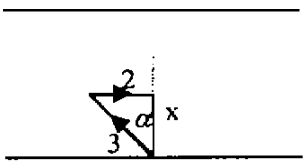
$|\vec{V}_{AB}| = \sqrt{15^2 + 20^2} = 25$ km/hr — (5)

$\tan\alpha = \frac{20}{15}$ — (5) — **20**

(b)

(i) **Shortest time** $= \frac{100}{3}$ — 10

(ii)

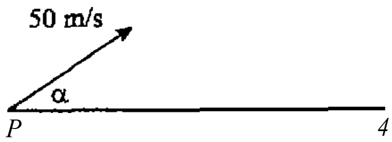


Direction: $\sin\alpha = \frac{2}{3}$ — 10

$x = \sqrt{9-4} = \sqrt{5}$

Time $= \frac{100}{\sqrt{5}}$ — 10 — 30

3. A particle is projected from a point p on level horizontal ground with an initial speed of 50 m/s inclined at an angle a to the horizontal where $\tan\alpha = \frac{3}{4}$.
The particle strikes the ground at the pointy on the same horizontal level as p.



Find
- (i) the maximum height reached by the particle
- (ii) the time of flight
- (iii) $|pq|$, the distance from p to q.

(i)

initial vertical velocity $= 50\sin\alpha = 30$ 5

maximum height

$$v^2 = u^2 + 2as$$

$$0 = 30^2 \ t \ \ 2(-10)s$$ 10

$$s = 45$$ 5 20

(ii)

Vertical displacement $= 0$ 5

$$30t - 5t^2 = 0$$ 5

$$t = 6$$ 5 15

c I

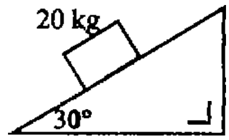
(iii)

$$|pq| = 50\cos\alpha . t$$ 10

$$= 40(6)$$

$$= 240m$$ 5 15

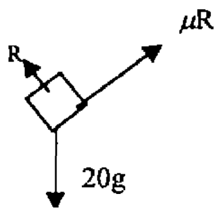
4 A particle of mass 20 kg is placed on a rough plane inclined at an angle 30° to the horizontal. The particle is on the point of moving down the plane.



(i) Show on a diagram all the forces acting on the particle.

(ii) Find the value of μ, the coefficient of friction between the particle and the plane.

Give your answer in the form $\frac{1}{\sqrt{p}}$, $p > 0$.



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$$R = 20g\cos 30$$ 5

$$\mu R = 20g\sin 30$$ 5

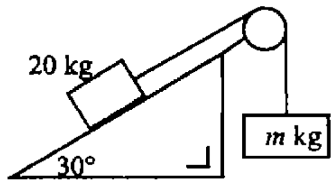
$$\mu(20g\cos 30) = 20g\sin 30$$

$$\mu = \tan 30 = \frac{1}{\sqrt{3}}$$ 5

25

4 cont.

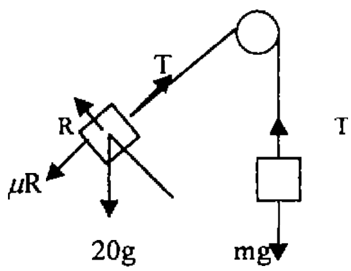
A smooth light pulley is now attached to the top of this plane. A particle of mass m kg. hanging freely under gravity, is now connected lo the particle of mass 20 kg by means of a light inextensible string passing over this smooth pulley at the lop of the plane.



The particles are released fmm rest.

The 20 kg particle moves with an acceleration of 2 m/s^2 up the plane.

Find the value of m and the value of the tension in the string.



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$$T - \mu R - 20g\sin 30 = 20(2)$$ 5

$$T - \frac{1}{\sqrt{3}}(20g\cos 30) - 20g\sin 30 = 20(2)$$

$$T - 100 - 100 = 40$$

$$T = 240 \text{ N}$$ 5

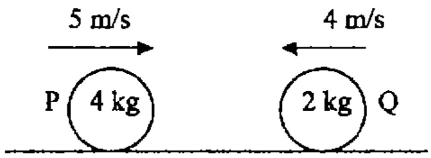
$$mg - T = m(2)$$

$$10m - 240 = 2m$$

$$m = 30 \text{ kg}$$ 5

25

5 Two smooth spheres P and Q, of masses 4 kg and 2 kg respectively and travelling in opposite directions with speeds of5 m/s and 4 m's respectively, collide directly on a smooth horizontaltable.
The coefficient of restitution between the spheres is e.
As a result of the collision P continues to move in the same direction with a speed of e m/s.



(i) Find the value of e .
(ii) Find the loss in kinetic energy due to the collision.

(i)

PCM $4(5)+2(-4) = 4v_1 + 2v_2$ — 10

$12 = 4e + 2v_2$

NEL $v_1 - v_2 = -e(5-(-4))$ — 10

$e - v_2 = -9e$

$v_2 = 10e$ — 5

$12 = 4e + 2(10e)$ — 5

$= 24e$

$e = \frac{1}{2}$ — 5

35

(ii)

KE before $= \frac{1}{2}(4)(5)^2 + \frac{1}{2}(2)(-4)^2$ — 5

$=$ SO + 16

$= 66$

KE after $= \frac{1}{2}(4)(\frac{1}{2})^2 +$ f(2)(5)' — 5

$= \frac{1}{2} + 25$

$= 25.5$

Loss in KE $= 66 - 25.5$

$= 40.5$ — 5

15

6. **(a)** Particles of weight 2 N, 3 N. 4 N and 1 N are placed at the points (−1, 2), (3. 1). (4. -3) and (2.5) respectively.

Find the coordinates of the centre of gravity of the four particles.

$10\bar{x} = 2(-1) + 3(3) + 4(4) + 1(2)$ 10

$\bar{x} = 2.5$ 5

$10\bar{y} = 2(2) + 3(1) + 4(-3) + 1(5)$ 10

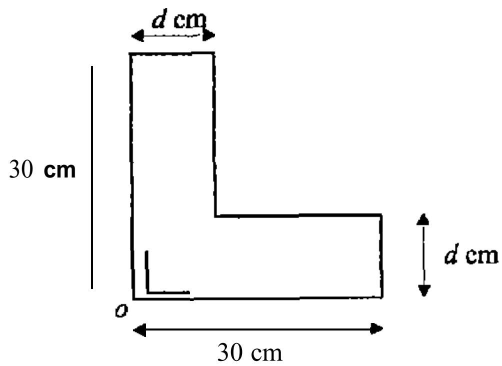
$\bar{y} = 0$ 5

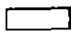
30

(b) A manufacturer makes the capital letter L from uniform plastic material with dimensions as shown in the diagram.

Taking the point *o* as the origin. the centre of gravity of the L shape is at the point (11, 11), in units of cm.

Calculate the value of *d*.



	area	cords of cg	
	30d	$\left(\frac{d}{2}, 15\right)$	5
	$(30-d)d$	$\left(d+\frac{30-d}{2}, \frac{d}{2}\right)$	5
	$60d - d^2$	$(11, 11)$	5

$$\left(60d-d^2\right)11 = \left(30d-d^2\right)\frac{d}{2} + 30d(15)$$

$$d^2 - 52d + 420 = 0$$

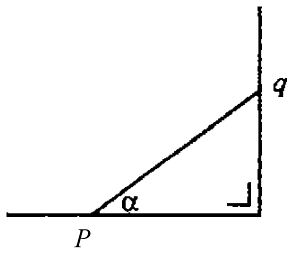
$$(d-10)(d-42) = 0$$

$$d = 10\text{cm}$$

5

20

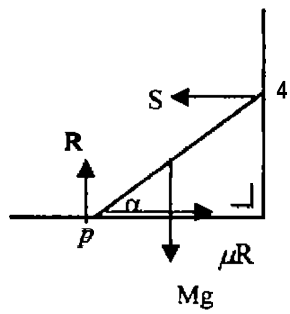
7. A uniform ladder, [pq], of mass M kg and length 5 m, has end p on rough dry horizontal ground and end q against a smooth vertical wall. The. ladder is on the point of slipping when inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$



(i) Show on a diagram all the forces acting on the ladder.

(ii) Find the value of μ , the coefficient of friction between the ladder and the ground.

(i)



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(ii)

Momentsabout p :

$$S\ (5\sin\alpha) = Mg(2.5\cos\alpha)$$ 10

$$S(2)\tan\alpha = Mg$$

$$S = \frac{2Mg}{3}$$ 5

$$S = \mu R$$ 5

$$\frac{2Mg}{3} = \mu Mg$$

$$\mu = \tfrac{2}{3}$$ 5

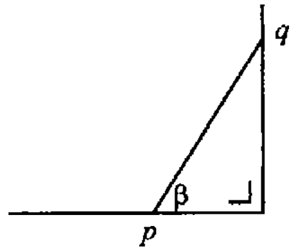
35

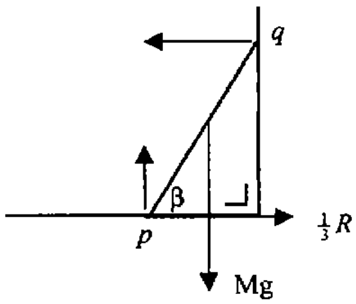
7 **cont.**

When the ground is wet. the ladder [pq] is on the point of slipping when inclined at an angle β to the horizontal, leaning against the same wall.
In this case, the value of μ is half the value it had when the ground was dry.

Show that

$$\tan \beta = \frac{3}{2} .$$





$$R = Mg$$

$$S = \tfrac{1}{3}R = \tfrac{1}{3}Mg$$ 5

Moments about p :

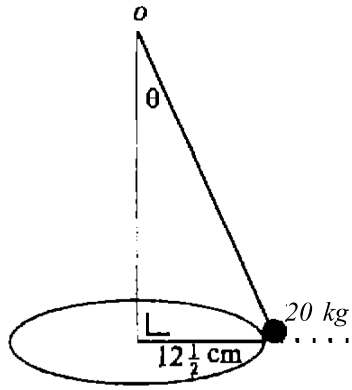
$$S\ (5\sin\beta) = Mg(2.5\cos\beta)$$ 5

$$\tfrac{1}{3}Mg(2)\tan\beta = Mg$$

$$\tan\beta = \frac{3}{2}$$ 5

15

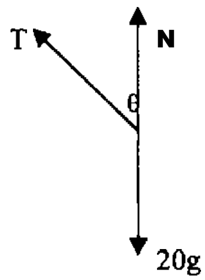
8. A particle of mass 20 kg describes a horizontal circle of radius length $12\frac{1}{2}$ cm with constant angular velocity of 4 rad/s on a smooth horizontal table. The particle is connected by means of a light inextensible string to a fixed point o which is vertically above the centre of the circle.
The inclination of the string to the vertical is θ, where $\tan\theta = \frac{5}{12}$.



(i) Show on a diagram all the forces acting on the particle.

(ii) Show that the value of the normal reaction between the particle and the table is equal to the value of the tension in the string.

(i)



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(ii)

$T\sin\theta = mr\omega^2$ 10

$T\left(\frac{5}{13}\right) = 20(0.125)(4^2)$ 5

$T = 104$ 5

$T\cos\theta + N = 20g$ 10

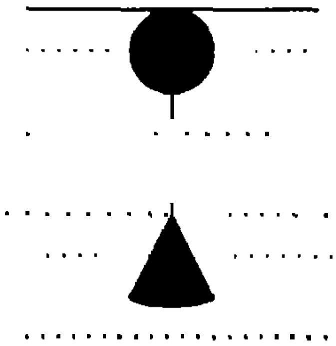
$104\left(\frac{12}{13}\right) + N = 200$

$N = 104$ 5

50

9. State the principle of Archimedes.

A solid sphere of radius length r cm is connected to a solid cone of base radius length r cm and height 2r cm by means of a light inelastic string. The density of the sphere is p kg/m^3 and the density of the cone is 2ρ kg/m^3.



The system is floating in a tank of water with the sphere just below the surface of the water.

(i) Show that the volume of the sphere is twice that of the cone.

Principle of Archimedes. | 10

(i)

Volume of sphere $= \frac{4}{3}\pi r^3$ | 5

Volume of cone $= \frac{1}{3}\pi r^2 h$ | 5

$= \frac{1}{3}\pi r^2(2r)$

$= \frac{2}{3}\pi r^3$ | 5

15

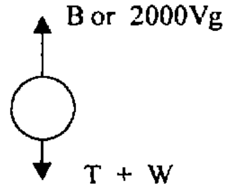
9cont.

(ii) Show, on separate diagrams. the forces acting on the sphere and on the cone.

(iii) Find the value of ρ

[Density of water = 1000 kg/ m^3]

(ii)



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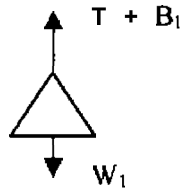

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(iii)

$$2000Vg - T = (2V)\rho g$$ 5

$$T + 1000Vg = V(2\rho)g$$ 5

$$3000Vg = 4V\rho g$$

$$\rho = 750$$ 5

25